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List of Frequently Used Symbols



UNIVERSIDAD NACIONAL DE ENTRE RIOS FACULTAD DE INGENIERIA CENTRO DE MEDIOS BIBLIOTECA

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