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Supplementary Chapters—Available from the authors as they are produced

S-1 Lie Groups

S-2 Introduction to Differential Topology

S-3 Topics in Riemannian Geometry