

UNIVERSIDAD NACIONAL D'ENTRE RIOS
FACULTAD DE IN ENIERIA
CENTRO DE MEDICS
BIBLIOTECA

1789

Contents

Pret	face	xiii
Part	1 First Thoughts on Equilibria and Stability	1
Cha	pter One Simple Dynamic Models	3
1.1	Back and Forth, Up and Down The spring-mass system in vertical and horizontal motion, with and without damping.	3
1.2		6
1.3	Stable Equilibria, I Reduction of second-order equations to first-order systems. Concepts of stable and unstable equilibria. Examples.	8
1.4	What Comes Out Is What Goes In Modeling problems involving conservation of mass.	12
1.5	Exercises	13
Cha	pter Two Stable and Unstable Motion, I	17
2.1	The Pendulum Formulation of our first nonlinear model, the pendulum.	17
2.2		19
		vii

	vector system $\dot{\mathbf{x}} = A\mathbf{x}$ in \mathbf{R}^2 . A necessary and sufficient condition is that the real parts of the eigenvalues of A be nonpositive.	
2.3	When Is a Nonlinear System Stable?	22
	Linearization of nonlinear first-order equations and the local stability of equilibria based on the ideas of 2.2. Examples, including the pendulum.	
2.4	The Phase Plane	26
	Models in which a conservation of energy relation is valid. Determining orbits in the p, \dot{p} phase plane without the need to explicitly solve the equation. Examples, including the undamped pendulum.	
2.5	Exercises	36
Cha	pter Three Stable and Unstable Motion, II	39
3.1	Liapunov Functions	39
0.1	A proof of Liapunov's theorem in R ² . Application to dissipative systems. Examples, including the damped pendulum.	00
3.2	Stable Equilibria, II	48
	An application of Liapunov's theorem to justify the use of linearization to determine local stability. An example shows that linearization fails to be useful for nonhyperbolic equilibria. Comments on the idea of robust (structurally stable) systems and the stability dogma.	
3.3	Feedback	52
	How to control the unstable equilibrium of the pendulum by feedback. This leads to a linearized system. A simple proof in \mathbb{R}^2 of the eigenvalue placement theorem allows a judicious choice of control.	
3.4	Exercises	58
01	* *	04
Cha	apter Four Growth and Decay	61
4.1	The Logistic Model The logistic equation formulated, solved, and interpreted. Stability analysis of its equilibria. Variants of the model.	61
4.2	Discrete Versus Continuous	66
7.2	The pitfalls of modeling continuous phenomena by difference equations and vice-versa.	00
4.3	The Struggle for Life, I	68
	The quadratic population model with special cases of predation, competi- tion, and combat. The method of isoclines.	
4.4		74
2 4	Linearization and analysis of equilibria of models in 4.3. Use of Liapunov functions.	
4.5	Exercises	78
Α .	Summary of Part 1	81

	Contents	ix
Part	2 Further Thoughts and Extensions	83
Cha	pter Five Motion in Time and Space	85
Chapter rive Wottom in Time and Space		
5.1	Conservation of Mass, II Derivation of a basic partial differential equation modeling the flow of a substance in a single spatial dimension over time. Applications in the next four sections.	85
5.2	Algae Blooms A model for the growth of algae. The minimum spatial dimension necessary to maintain a sustained population is obtained by separation of variables. Boundary conditions. The use of phase plane methods.	89
5.3	Pollution in Rivers Coupled linear equations for oxygen depletion in a river due to pollutants. Steady state solutions. Traveling wave solutions.	95
5.4	Highway Traffic A model for the flow of traffic along a highway. An introduction to characteristics and the propagation of shocks. More on traveling wave solutions. Burger's equation.	101
5.5	A Digression on Traveling Waves Comments on traveling wave solutions to Fisher's equation.	111
5.6	Morphogenesis A reaction-diffusion model for morphogenesis. A uniform equilibrium distribution of the cells can become unstable thereby leading to a spatially nonhomogeneous pattern. A similar model is discussed that suggests patchy growth of algae in an ocean.	115
5.7	Tidal Dynamics The movement of water in estuaries and canals due to ocean tides leads to a pair of nonlinear equations via the principle of conservation of	125
	momentum. Traveling wave solutions.	
5.8	Exercises	131
Cha	pter Six Cycles and Bifurcation	137
6.1	Self-Sustained Oscillations The spring-mass system of 1.1 is re-examined under Coulomb damping on a moving surface. This leads to limit cycles in a model of a bow moving across a violin string or a brake pad against a moving wheel rim.	137
6.2	When Do Limit Cycles Exist? Positive limiting sets of an orbit. Statement and explanation of Poincaré— Bendixson theorem (no proof). Simple examples lead to the bifurcation of an orbit from a stable equilibrium to a stable cycle. A heuristic proof is given of the Hopf bifurcation theorem in the plane.	143
6.3	The Struggle for Life, II A more general model of predation, which includes satiation and a model of harvesting fish stocks. Both models lead to limit cycles.	155

6.4	The Flywheel Governor Formulation of our first model with three equations, the Watt governor. When the equilibrium is unstable, a limit cycle develops as a Hopf bifurcation.		
6.5	Exercises	167	
Cha	pter Seven Bifurcation and Catastrophe	171	
7.1	Fast and Slow In some models, certain variables undergo rapid change as certain other parameters vary slowly. We consider one or two parameters. Potential functions and gradient systems. Heuristic treatment (no proof) of Thom's theorem on fold and cusp catastrophes. Relation to bifurcation. The idea of resilience. Applications in the next three sections.	171	
7.2	The Pumping Heart The Zeeman model of the heart is formulated as a function of stimulus and tension.	182	
7.3	Insects and Trees The Holling-Ludwig-Jones model of budworm infestation of spruce forests as a function of branch size and foliage.	189	
7.4	The Earth's Magnet A modified Bullard model of the earth's magnetic field, leading to field reversals.	196	
7.5	Exercises Including a model of algae bloom as a function of nutrient level and tidal flow.	202	
Cha	pter Eight Chaos	207	
8.1	Not All Attractors Are Limit Cycles or Equilibria We begin our study of models that display erratic behavior with the Leonard-May model of competition between three groups of participants.	207	
8.2	Strange Attractors The chaotic behavior of a modified version of the geomagnetic equations.	214	
8.3	Deterministic or Random? The discrete logistic equation displays apparently random behavior. This is explained on the basis of symbolic dynamics.	218	
8.4	Exercises	227	
Cha	pter Nine There Is a Better Way	229	
9.1	Conditions Necessary for Optimality The maximum principle of Pontryagin is stated in a form sufficient to handle a number of applications. We begin by reconsidering the stabilization of the inverted pendulum.	229	
9.2	Fish Harvesting The turnpike theorem and a model of optimal exploitation of renewable	236	

			Contents	xi
9.3	Bang – Bang Controls The harmonic oscillator is re-examined as an optimization problem. Also considered is the problem of optimal rocket flight.		243	
9.4	Exercises			
App		rdinary Differential Equations: A ne basic facts about differential equation		255
	First-Order	Equations (The Case $k = 1$)		256
	The Case	c = 2		257
	The Case	c = 3		261
Ref	erences and	a Guide to Further Readings		263
	Ordinary D	ifferential Equations		264
		ons to Differential Equation Mode	eling	264
		nced Modeling Books		265
	Hard to Cla	•		266
Not	es on the Inc	dividual Chapters		267
Inde	ex			275



