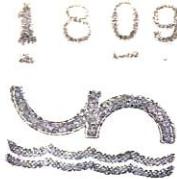


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UNIVERSIDAD NACIONAL DE ENTRE RÍOS  
FACULTAD DE INGENIERIA  
CENTRO DE MEDIOS  
BIBLIOTECA

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