Table of Contents

FOREWORD v

1 SUMMATIONS 1
Summation of a Constant, Discrete Data 2
Summation of a Constant Times a Variable, Discrete Data . . 3
Product of a Series of Constants 3
Product of a Constant Times a Variable 4
Double and Triple Sums 5
The Sum of a Special Product 7
The Sum $\Sigma_i^a n_i (\bar{x}_i - \mu)$ 8
Integral of a Constant 9
Integral of a Variable 9
Sum of the First n Positive Integers 10
Sum of the Squares of the First n Positive Integers . . . 11

2 PROBABILITY 12
Basic Concepts of Discrete Probability 13
Basic Concepts of Continuous Probability 16
Addition of Probabilities 17
Conditional Probability 20
Independence 22
Bayes' Rule 23

3 EXPECTED VALUE 25
Mathematical Expectation, Basic Definition 27
Expected Value of a Constant 27
Expected Value of a Discrete Random Variable 28
Expected Value of a Sum of Two Discrete Random Variables. . 29
Expected Value of the Mean of a Discrete Random Variable . . 29
Expectation of a Product, Discrete Independent Random Variables 30
Expectation of a Continuous Random Variable 31
Expected Value of the Sampling Variance 32
Sampling Variance, Grouped Data 34
Expected Value of the Variance of the Sampling Mean . . . 35

4 SOME APPLICATIONS OF EXPECTED VALUES . . . 37
Variance of a Difference 38
Variance of the Difference Between Means 38
Variance of the Mean for a Finite Population 39
Covariance of Two Random Variables, Finite Populations . . 41

5 PROBABILITY DISTRIBUTIONS 44
Binomial Probability Mass Function 48
Hypergeometric Probability Mass Function 49
Poisson Probability Mass Function 50
Exponential Probability Density Function 52
Normal Curve Probability Density Function 54
Chi-Square Probability Density Function 58
Student-t Probability Density Function 58
F-Distribution 61
Cauchy Probability Density Function. 64
Weibull Probability Density Function. 66

6 MOMENTS OF A DISTRIBUTION 68
Moment Generating Functions, General Theory 69
Moment Generating Function of the Binomial Distribution, an Example of Specific Application of the General Theory . . 70
Moments of the Binomial Distribution 71
Moment Generating Function for the Discrete Uniform Distribution 74
Moments of the Continuous Uniform Distribution . . . 75
Moments of the Poisson Distribution 76
Moments of the Exponential Distribution 78
Moments of the Chi-Square Distribution 80
Moments of the Hypergeometric Distribution 83
Moments of the Normal Distribution 85
Moment of the Cauchy Distribution (Proof of their Non-Existence). 88
Moments of the Weibull Distribution 89
Moments of the Gamma Distribution 91
Moments of the Beta Distribution 93

7 MAXIMUM LIKELIHOOD FUNCTIONS 95
Maximum Likelihood Estimate of Mean, Normal Distribution. . 96
Maximum Likelihood Estimate of Variance, Normal Distribution . 97

8 ANALYSIS OF VARIANCE 99
One-Way Classification Variance Analysis, Fundamental Identity . 100
One-Way Classification Variance Analysis Fixed Effects Model, Expected Mean Squares 102
One-Way Classification Variance Analysis, Random Effects Model, Expected Mean Squares 105
Two-Way Classification Variance Analysis, One Observation Per Cell, Fundamental Identity 108
Two-Way Classification Variance Analysis, One Observation Per Cell, Random Effects Model, Expected Mean Squares . . 110

Two-Way Classification Replicated Variance Analysis, Random Effects Model, Expected Mean Squares 114
Sum of Squares, Calculation Formulas 121

9 LINEAR REGRESSION CORRELATION 124
Maximum Likelihood Estimates for Simple Linear Regression. . 126
Unbiasedness of Estimators $\hat{\alpha}$ and $\hat{\beta}$ in Regression. 128
Unbiased Estimator of the Variance in Simple Linear Regression . 131
Sufficient Conditions for Minimum Squared Deviations in Simple Regression Analysis 134
Correlation Analysis 136

10 BAYESIAN ANALYSIS 139
The Mean of the Posterior Distribution 140
The Variance of the Posterior Distribution. 144
Posterior Distribution 144

11 SERIES, TRANSFORMATIONS AND ADDITIONAL FUNCTIONS 146
Taylor's Series 147
MacLaurin's Series 148
Jacobian Transformation 148
Hessian Transformation 149
Gamma Function 150
Beta Distribution 152
Beta Distribution Obtained from the F-Distribution . . . 154

APPENDIX A, Differentiation Formulas 157

APPENDIX B, Integration Formulas 159

APPENDIX C, Fundamental Trigonometric Relations . . . 161

APPENDIX D, Selected Topics from Algebra 164

APPENDIX E, Problems 169

APPENDIX F, Solutions to Problems Provided in Appendix E . . 177