CONTENTS

№ 2705

Chapter 1 **BASIC DEFINITIONS, DETERMINANTS, AND LINEAR ALGEBRAIC EQUATIONS 1**

1.1. Introduction 1
1.2 Determinants of the Second Order 1
1.3 Properties of Determinants 2
1.4 Multiplication of Determinants. 2
1.5 Two Simultaneous Linear Algebraic Equations in Two Unknowns 3
1.6 Determinants of the *n*th Order 4
1.7 Expansion of Determinants of the *n*th Order . . 5
1.8 Fundamental Properties of Determinants . . . 6
1.9 Method of Chiò for Evaluating Determinants . . 8
1.10 The Solution of Linear Equations by Determinants, Cramer's Rule 12
1.11 *n* Homogeneous Equations in *n* Unknowns . . 14
1.12 Gauss' Method of Elimination in Solving Simultaneous Equations 15
1.13 Triangulation Method in the Solution of Simultaneous Equations 16

Chapter 2 **MATRIX ALGEBRA, SIMULTANEOUS EQUATIONS 22**

2.1 Introduction 22
2.2 Definitions and Notation 22
2.3 Elementary Operations on Matrices 23
2.4 Multiplication of Matrices 24
2.5 Partitioned Matrices and Partitioned Multiplication . 26
2.6 Matrix Division; the Inverse Matrix 28
2.7 Solution of Simultaneous Equations by Matrix Inversion 31
2.8 Augmented Matrix Method of Matrix Inversion . 32
2.9 Gauss-Seidel Method of Solving Simultaneous Equations 38
2.10 Representation of Vectors by Matrices . . . 41
2.11 Coordinate Transformation Using Matrices . . 43

2.12 The Vector Product of Two Vectors 44
2.13 Linear Dependence of Vectors 46
2.14 Linear Independence of Vectors and the Gram Determinant 48

Chapter 3 **EIGENVALUES, EIGENVECTORS, AND QUADRATIC FORMS** 55

3.1 Introduction 55
3.2 The Eigenvector (Characteristic Vector) Equation . 55
3.3 Fundamental Properties of the Characteristic Polynomial of Matrix A 59
3.4 Properties of the Eigenvectors of a Square Matrix . 62
3.5 The Case of Repeated Eigenvalues 66
3.6 Matrix Iteration Method of Evaluating Eigenvalues and Eigenvectors 69
3.7 Iteration Method for Obtaining the Smallest Characteristic Values and Vectors 76
3.8 Use of Orthogonality Relations in Obtaining Characteristic Values and Vectors 77
3.9 The Method of A. M. Danilevsky for Obtaining the Characteristic Equation 81
3.10 Characteristic Vector by the Method of A. M. Danilevsky 88
3.11 The Relation Between the Eigenvectors and Eigenvalues of a Matrix and Those of Its Transpose . . 89
3.12 The Gram-Schmidt Orthogonalization Process . . 90
3.13 The Modal Matrix and the Quadratic Forms . . 91

Chapter 4 **FUNCTIONS OF MATRICES; MATRIX CALCULUS AND MATRIX DIFFERENTIAL EQUATIONS** 99

4.1 Introduction 99
4.2 Polynomials in a Square Matrix, with Scalar Coefficients 99
4.3 Power Series of Matrices 100
4.4 Matrix Functions 101
4.5 Some Properties of the Matrix Exponential Function . 101
4.6 Rational Functions of Matrices 103
4.7 Eigenvalues of Rational Functions of Matrices (Frobenius's Theorem) 103
4.8 The Cayley-Hamilton Theorem 104
4.9 The Reduction of Matrix Polynomials 106
4.10 Reduction of Functions of Matrices 107
4.11 Reduction of Functions of *n*th order Matrices . . 110
4.12 Reduction of Polynomials and Functions of Matrices with Multiple Eigenvalues 112

4.13 Differentiation and Integration of Matrices . . 112
4.14 Simultaneous Linear Differential Equations of the First Order with Constant Coefficients 114
4.15 Solution of a Homogeneous Set of Linear Differential Equations of the Second Order 116
4.16 Simultaneous Linear Differential Equations of the First Order with Variable Coefficients 117
4.17 The Determination of the Transition Matrix by the Method of Picard 119
4.18 Computation of the Transition Matrix $P(t)$ . . 122
4.19 Certain Numerical Methods for the Determination of the Transition Matrix $P(t)$ 123
4.20 The Method of Mean Coefficients for Evaluating $P(t)$. 127
4.21 A Perturbation Method for the Determination of the Transition Matrix $P(t)$ 129
4.22 A Method for the Computation of the Transition Matrix $P(t)$ for a Constant Matrix A 131

Chapter 5 **ELECTRICAL APPLICATIONS OF MATRICES 141**

5.1 Introduction 141
5.2 Fundamental Principles 141
5.3 The General Circuit on a Loop Basis 143
5.4 The Laplace Transform 146
5.5 The General Solution of the Loop Equations . . 150
5.6 The Single-Loop Case 152
5.7 The Steady-State Solution of the n-Loop Circuit; Alternating Currents 155
5.8 The Nodal Equation of the General Network . . 156
5.9 Four-Terminal Networks in the Alternating Current Steady State 160
5.10 Reversed Four-Terminal Networks 162
5.11 The Series Interconnection of Four-Terminal Networks 163
5.12 Wave Propagation Along a Series of Four-Terminal Networks 166
5.13 Wave Propagation Along a Series of Reversible Structures 170
5.14 Elementary Filter Circuits 172
5.15 Steady-State Analysis of Two-Conductor Transmission Lines 176

Chapter 6 **TIME-FREQUENCY DOMAIN ANALYSES AND THE FAST FOURIER TRANSFORM 182**

6.1 Introduction 182
6.2 Discrete Fourier Transform 183
6.3 Matrix Calculation of Fourier Transforms . . . 187
6.4 Numerical Examples 190
6.5 Computer Program for the Calculation of the Fourier Transform 194
6.6 Cooley-Tukey Method 197
6.7 Discussion of Fourier Transforms 201
6.8 Electric Circuit Analysis in the Time Domain . . 202
6.9 The Free Oscillations of the Circuit 203
6.10 Solution in Terms of Functions of Matrices . . 206
6.11 The General Case, Forced Oscillations . . . 208

Chapter 7 **FORMULATION OF VIBRATION PROBLEMS (CONSERVATIVE SYSTEMS) 213**

7.1 Introduction 213
7.2 System with One Degree of Freedom 213
7.3 Free Vibrations of Systems with Two Degrees of Freedom 217
7.4 The Free Oscillations of a Linear Conservative System with n Degrees of Freedom 225
7.5 Normal Coordinates in the Case of n Degrees of Freedom 229
7.6 Examples of the General Theory 230
7.7 The Modal Matrix and the Normal Coordinates. . 233
7.8 The Case of Zero Frequency 235
7.9 Use of Matrix Iteration in Determining the Frequencies and Modes of Oscillation of Linear Conservative Systems 238
7.10 Numerical Example of the Iteration Procedure . . 241
7.11 Determination of the Higher Modes; the Sweeping Matrix 244
7.12 Numerical Example: the Triple Pendulum . . . 247

Chapter 8 **FORMULATION OF VIBRATION PROBLEMS (NONCONSERVATIVE SYSTEMS) 258**

8.1 Introduction 258
8.2 The Reduced Equations 259
8.3 Separated Coordinates 262

8.4 The Forced Oscillations 262
8.5 The Analysis of a Class of Symmetric Damped Linear Systems 263
8.6 Matrix Iteration Method for Damped Vibrations. . 265
8.7 Determination of the Frequencies and Modes . . 267
8.8 Completion of the Solution 271
8.9 Transformation of the Equations of Motion and Normal Coordinates 273
8.10 Remarks on Numerical Procedures 274
8.11 Numerical Example 274
8.12 The Routh-Hurwitz Stability Criterion 280

Chapter 9 **STRUCTURAL APPLICATION OF MATRICES 284**

9.1 Introduction 284
9.2 Stiffness and Flexibility Matrices 284
9.3 Equations of Motion 292
9.4 Internal Forces, Deformation, and Internal Energy . 294
9.5 Matrix Force Method of Calculating Stiffness and Flexibility Matrices 297
9.6 Basic Theory of Matrix Force Method 297
9.7 Method of Computing Matrix X 304
9.8 Procedure for the Calculation of Internal Forces, Deformations, and Flexibility Matrix Using Matrix Force Method 306
9.9 Displacement Method of Calculating the Stiffness Matrix 313
9.10 Partitioning of the Stiffness Matrix 318
9.11 Analogy between the Force Method and the Displacement Method 322

INDEX 329