Table of Contents

LIST OF FIGURES ix

LIST OF TABLES xi

SERIES PREFACE xiii

PREFACE xv

1. INTRODUCTION 1
- A. Statement of the Problems 2
- B. Research Objectives 3

2. LITERATURE REVIEW 4
- A. Direct Kinematics 4
- B. Inverse Kinematics 6
- C. Differential Motions 8

3. DIRECT MANIPULATOR KINEMATICS 10
- A. Mathematical Preliminaries 10
  - 1. Translation Vector 10
  - 2. Rotation (Orientation) Matrix 12
  - 3. Homogeneous Transformation 14
  - 4. Transformations 15
  - 5. Composite Homogeneous Transformation 16
  - 6. Premultiplication and Postmultiplication of the Homogeneous Transformation Matrices 17
  - 7. Inverse Homogeneous Transformation 18
- B. Mathematical Modeling of the Manipulator 19
  - 1. Denavit-Hartenberg Representation 19
  - 2. Specification of **A** Matrices 21
  - 3. Specification of the $\mathbf{T}_n$ Matrix 22
- C. Kinematic Control Equations 24
- D. Wrist Kinematic Control Equation 25
  - 1. Euler Angles 25
  - 2. Specification of Euler Angles in Terms of the **A** Matrices 27

3. Roll, Pitch, and Yaw 28
4. Specification of Roll, Pitch, and Yaw Angles in Terms of the A Matrices 29
E. Arm Kinematic Control Equation 32
1. Cartesian Coordinate Arm (PPP) 33
2. Cylindrical Coordinates 35
3. Cylindrical Coordinate Arm (RPP) 36
4. Spherical Coordinates 39
5. Spherical Coordinate Arm (RRP/PRR) 39
6. Articulated Coordinate Arm (RRR) 41
F. Direct Kinematics Solutions 43
1. Spherical Arm with Euler Wrist 44
2. Articulated Arm with Roll-Pitch-Yaw Wrist 48

4. DIFFERENTIAL MOTIONS 52
A. Relative Motions Between Frames 52
1. Angular and Linear Velocities of the Prismatic Joint 55
2. Angular and Linear Velocities of the Revolute Joint 56
3. Angular and Linear Velocities of the Prismatic/Revolute Joint with Respect to the Base Frame 59
4. Angular and Linear Velocities of the Prismatic/Revolute Joint with Respect to the End Effector Frame 61
B. Formulating the Manipulator Jacobian Matrix 63
1. The Jacobian Matrix with Respect to the Base Frame 66
2. The Jacobian Matrix with Respect to the End Effector Frame 67
3. The Relation between the Jacobian Matrices of the Base Frame and the End Effector Frame 67
C. Solution for the Arm Jacobian 68
1. Jacobian of the Cylindrical Arm 68
2. Jacobian of the Spherical Arm 69
D. Solution for the Wrist Jacobian 70
1. Jacobian of the Euler Wrist 70
2. Jacobian of the Roll-Pitch-Yaw Wrist 71
E. Solution for the Manipulator Jacobian 71
F. Speed Control and the Pseudo-Inverse Jacobian Matrix 75
1. The Overdetermined Case 75
2. The Underdetermined Case 76

5. SYMBOLICALLY AUTOMATED DIRECT KINEMATICS AND JACOBIAN EQUATIONS SOLVER 79
A. Direct Kinematics Equations 79
1. Backward and Forward Recursive Symbolic Relations 80

2. Backward Recursive Equations for a Revolute Joint 81
3. Backward Recursive Equations for a Prismatic Joint 83
4. Revolute Joints with Parallel Axes 84
B. Manipulator Jacobian Matrix 85
1. Recursive Symbolic Equations for the Jacobian with Respect to the End Effector Frame 86
C. PROLOG 87
1. Terms and Clauses 87
2. Prolog Control Structure 89
3. Knowledge Representation 90
4. Inference Mechanism 91
D. Automated Link Transformation Matrices Generator 91
1. Knowledge in Facts 91
2. Knowledge in Production Rules 93
E. Symbolically Automated Direct Kinematics Problems Solver 96
1. Knowledge about the Multiplication Rules 97
2. Knowledge about the Trigonometric Identities 100
3. Heuristic Rules for Simplifying the Expression 101
F. Symbolically Automated Jacobian Matrix 104
G. Example 104
H. Conclusion 108

6. INVERSE MANIPULATOR KINEMATICS 109
A. Inverse Homogeneous Transformation Matrices Approach 109
1. Euler Wrist 110
2. Roll-Pitch-Yaw Wrist 114
3. Cartesian Arm 115
4. Cylindrical Arm 116
5. Spherical Arm 117
6. Articulated Arm 118
7. Articulated Arm with Roll-Pitch-Yaw Wrist 121
B. Geometric Approach 125
1. Inverse Kinematics Solution of the Jumbo Drilling Robot 126
C. Arm-Wrist Partitioned Synthesis Approach 130
1. Nomenclature and Problem 131
2. Solution of the Arm Synthesis 134
3. Solution of the Wrist Synthesis 174
4. Solution of the Arm-Wrist Partitioned Synthesis 182
D. Numerical Examples 183
1. Cincinnati T3-756 Robot 183
2. Jumbo Drilling Robot 188

7. RESULTS AND CONCLUSIONS 192
A. Results 192

B. Analysis 219
1. Symbolically Automated System 219
2. Arm-Wrist Partitioned Synthesis 220
C. Conclusions 226
D. Future Research 228

REFERENCES 230

APPENDICES

A. Solutions of Algebraic Equations 233
B. Program Listing for the Symbolically Automated System 235
C. Program Listing for the Arm-wrist Partitioned Synthesis Algorithm 268

AUTHOR INDEX 311

SUBJECT INDEX 313