



Contents

PREFACE	xiv
1 Introduction	1
1-1 Mathematical Representation of Signals	2
1-2 Mathematical Representation of Systems	4
1-3 Thinking About Systems	5
1-4 The Next Step	6
2 Sinusoids	7
2-1 Tuning Fork Experiment	8
2-2 Review of Sine and Cosine Functions	9
2-3 Sinusoidal Signals	11

2-3.1	Relation of Frequency to Period	12
2-3.2	Phase Shift and Time Shift	13
2-4	Sampling and Plotting Sinusoids	15
2-5	Complex Exponentials and Phasors	17
2-5.1	Review of Complex Numbers	17
2-5.2	Complex Exponential Signals	18
2-5.3	The Rotating Phasor Interpretation	19
2-5.4	Inverse Euler Formulas	21
2-6	Phasor Addition	22
2-6.1	Addition of Complex Numbers	23
2-6.2	Phasor Addition Rule	23
2-6.3	Phasor Addition Rule: Example	24
2-6.4	MATLAB Demo of Phasors	25
2-6.5	Summary of the Phasor Addition Rule	26
2-7	Physics of the Tuning Fork	27
2-7.1	Equations from Laws of Physics	27
2-7.2	General Solution to the Differential Equation	29
2-7.3	Listening to Tones	29
2-8	Time Signals: More Than Formulas	29
2-9	Summary and Links	30
2-10	Problems	31

3 Spectrum Representation 36

3-1	The Spectrum of a Sum of Sinusoids	36
3-1.1	Notation Change	38
3-1.2	Graphical Plot of the Spectrum	38
3-2	Beat Notes	39
3-2.1	Multiplication of Sinusoids	39
3-2.2	Beat Note Waveform	40
3-2.3	Amplitude Modulation	41
3-3	Periodic Waveforms	43
3-3.1	Synthetic Vowel	44
3-3.2	Example of a Nonperiodic Signal	45
3-4	Fourier Series	47
3-4.1	Fourier Series: Analysis	48
3-4.2	Fourier Series Derivation	48
3-5	Spectrum of the Fourier Series	50

3-6	Fourier Analysis of Periodic Signals	51
3-6.1	The Square Wave	52
3-6.1.1	DC Value of a Square Wave	53
3-6.2	Spectrum for a Square Wave	53
3-6.3	Synthesis of a Square Wave	54
3-6.4	Triangle Wave	55
3-6.5	Synthesis of a Triangle Wave	56
3-6.6	Convergence of Fourier Synthesis	57
3-7	Time–Frequency Spectrum	57
3-7.1	Stepped Frequency	59
3-7.2	Spectrogram Analysis	59
3-8	Frequency Modulation: Chirp Signals	60
3-8.1	Chirp or Linearly Swept Frequency	60
3-8.2	A Closer Look at Instantaneous Frequency	62
3-9	Summary and Links	63
3-10	Problems	64

4 Sampling and Aliasing 71

4-1	Sampling	71
4-1.1	Sampling Sinusoidal Signals	73
4-1.2	The Concept of Aliasing	75
4-1.3	Spectrum of a Discrete-Time Signal	76
4-1.4	The Sampling Theorem	77
4-1.5	Ideal Reconstruction	78
4-2	Spectrum View of Sampling and Reconstruction	79
4-2.1	Spectrum of a Discrete-Time Signal Obtained by Sampling	79
4-2.2	Over-Sampling	79
4-2.3	Aliasing Due to Under-Sampling	81
4-2.4	Folding Due to Under-Sampling	82
4-2.5	Maximum Reconstructed Frequency	83
4-3	Strobe Demonstration	84
4-3.1	Spectrum Interpretation	87
4-4	Discrete-to-Continuous Conversion	88
4-4.1	Interpolation with Pulses	88
4-4.2	Zero-Order Hold Interpolation	89
4-4.3	Linear Interpolation	90

4-4.4	Cubic Spline Interpolation	90
4-4.5	Over-Sampling Aids Interpolation	91
4-4.6	Ideal Bandlimited Interpolation	92
4-5	The Sampling Theorem	93
4-6	Summary and Links	94
4-7	Problems	96

5 FIR Filters 101

5-1	Discrete-Time Systems	102
5-2	The Running-Average Filter	102
5-3	The General FIR Filter	105
5-3.1	An Illustration of FIR Filtering	106
5-3.2	The Unit Impulse Response	107
5-3.2.1	Unit Impulse Sequence	107
5-3.2.2	Unit Impulse Response Sequence	108
5-3.2.3	The Unit-Delay System	109
5-3.3	Convolution and FIR Filters	110
5-3.3.1	Computing the Output of a Convolution	110
5-3.3.2	Convolution in MATLAB	111
5-4	Implementation of FIR Filters	111
5-4.1	Building Blocks	111
5-4.1.1	Multiplier	112
5-4.1.2	Adder	112
5-4.1.3	Unit Delay	112
5-4.2	Block Diagrams	113
5-4.2.1	Other Block Diagrams	113
5-4.2.2	Internal Hardware Details	115
5-5	Linear Time-Invariant (LTI) Systems	115
5-5.1	Time Invariance	116
5-5.2	Linearity	117
5-5.3	The FIR Case	117
5-6	Convolution and LTI Systems	118
5-6.1	Derivation of the Convolution Sum	118
5-6.2	Some Properties of LTI Systems	120
5-6.2.1	Convolution as an Operator	121
5-6.2.2	Commutative Property of Convolution	121
5-6.2.3	Associative Property of Convolution	121

5-7	Cascaded LTI Systems	122
5-8	Example of FIR Filtering	124
5-9	Summary and Links	126
5-10	Problems	126

6 Frequency Response of FIR Filters 130

6-1	Sinusoidal Response of FIR Systems	130
6-2	Superposition and the Frequency Response	132
6-3	Steady-State and Transient Response	135
6-4	Properties of the Frequency Response	137
	6-4.1 Relation to Impulse Response and Difference Equation	137
	6-4.2 Periodicity of $H(e^{j\omega})$	138
	6-4.3 Conjugate Symmetry	138
6-5	Graphical Representation of the Frequency Response	139
	6-5.1 Delay System	139
	6-5.2 First-Difference System	140
	6-5.3 A Simple Lowpass Filter	142
6-6	Cascaded LTI Systems	143
6-7	Running-Average Filtering	145
	6-7.1 Plotting the Frequency Response	146
	6-7.2 Cascade of Magnitude and Phase	148
	6-7.3 Experiment: Smoothing an Image	149
6-8	Filtering Sampled Continuous-Time Signals	151
	6-8.1 Example: Lowpass Averager	152
	6-8.2 Interpretation of Delay	154
6-9	Summary and Links	155
6-10	Problems	157

7 z -Transforms 163

7-1	Definition of the z -Transform	164
7-2	The z -Transform and Linear Systems	165
	7-2.1 The z -Transform of an FIR Filter	166

7-3 Properties of the z -Transform 167

7-3.1 The Superposition Property of the z -Transform 168

7-3.2 The Time-Delay Property of the z -Transform 168

7-3.3 A General z -Transform Formula 169

7-4 The z -Transform as an Operator 169

7-4.1 Unit-Delay Operator 169

7-4.2 Operator Notation 170

7-4.3 Operator Notation in Block Diagrams 170

7-5 Convolution and the z -Transform 171

7-5.1 Cascading Systems 173

7-5.2 Factoring z -Polynomials 174

7-5.3 Deconvolution 175

7-6 Relationship Between the z -Domain and the $\hat{\omega}$ -Domain 175

7-6.1 The z -Plane and the Unit Circle 176

7-6.2 The Zeros and Poles of $H(z)$ 177

7-6.3 Significance of the Zeros of $H(z)$ 178

7-6.4 Nulling Filters 179

7-6.5 Graphical Relation Between z and $\hat{\omega}$ 180

7-7 Useful Filters 181

7-7.1 The L -Point Running-Sum Filter 181

7-7.2 A Complex Bandpass Filter 183

7-7.3 A Bandpass Filter with Real Coefficients 185

7-8 Practical Bandpass Filter Design 186

7-9 Properties of Linear-Phase Filters 189

7-9.1 The Linear-Phase Condition 189

7-9.2 Locations of the Zeros of FIR Linear-Phase Systems 189

7-10 Summary and Links 190

7-11 Problems 191

8 IIR Filters 196

8-1 The General IIR Difference Equation 197

8-2 Time-Domain Response 198

8-2.1 Linearity and Time Invariance of IIR Filters 199

8-2.2 Impulse Response of a First-Order IIR System 200

8-2.3 Response to Finite-Length Inputs 201

8-2.4 Step Response of a First-Order Recursive System 202

8-3 System Function of an IIR Filter 204

8-3.1 The General First-Order Case 205

8-3.2	The System Function and Block-Diagram Structures	206
8-3.2.1	Direct Form I Structure	206
8-3.2.2	Direct Form II Structure	207
8-3.2.3	The Transposed Form Structure	208
8-3.3	Relation to the Impulse Response	209
8-3.4	Summary of the Method	209
8-4	Poles and Zeros	210
8-4.1	Poles or Zeros at the Origin or Infinity	211
8-4.2	Pole Locations and Stability	211
8-5	Frequency Response of an IIR Filter	212
8-5.1	Frequency Response using MATLAB	213
8-5.2	Three-Dimensional Plot of a System Function	214
8-6	Three Domains	216
8-7	The Inverse z -Transform and Some Applications	216
8-7.1	Revisiting the Step Response of a First-Order System	217
8-7.2	A General Procedure for Inverse z -Transformation	218
8-8	Steady-State Response and Stability	220
8-9	Second-Order Filters	223
8-9.1	z -Transform of Second-Order Filters	223
8-9.2	Structures for Second-Order IIR Systems	224
8-9.3	Poles and Zeros	225
8-9.4	Impulse Response of a Second-Order IIR System	226
8-9.4.1	Real Poles	227
8-9.5	Complex Poles	228
8-10	Frequency Response of Second-Order IIR Filter	231
8-10.1	Frequency Response via MATLAB	232
8-10.2	3-dB Bandwidth	232
8-10.3	Three-Dimensional Plot of System Functions	233
8-11	Example of an IIR Lowpass Filter	236
8-12	Summary and Links	237
8-13	Problems	238

9 Continuous-Time Signals and LTI Systems 245

9-1	Continuous-Time Signals	246
9-1.1	Two-Sided Infinite-Length Signals	246
9-1.2	One-Sided Signals	247
9-1.3	Finite-Length Signals	248

9-2	The Unit Impulse	248
9-2.1	Sampling Property of the Impulse	250
9-2.2	Mathematical Rigor	252
9-2.3	Engineering Reality	252
9-2.4	Derivative of the Unit Step	252
9-3	Continuous-Time Systems	254
9-3.1	Some Basic Continuous-Time Systems	254
9-3.2	Continuous-Time Outputs	255
9-3.3	Analogous Discrete-Time Systems	255
9-4	Linear Time-Invariant Systems	255
9-4.1	Time-Invariance	256
9-4.2	Linearity	256
9-4.3	The Convolution Integral	257
9-4.4	Properties of Convolution	259
9-5	Impulse Responses of Basic LTI Systems	260
9-5.1	Integrator	260
9-5.2	Differentiator	261
9-5.3	Ideal Delay	261
9-6	Convolution of Impulses	261
9-7	Evaluating Convolution Integrals	263
9-7.1	Delayed Unit-Step Input	263
9-7.2	Evaluation of Discrete Convolution	267
9-7.3	Square-Pulse Input	268
9-7.4	Very Narrow Square Pulse Input	269
9-7.5	Discussion of Convolution Examples	270
9-8	Properties of LTI Systems	270
9-8.1	Cascade and Parallel Combinations	270
9-8.2	Differentiation and Integration of Convolution	272
9-8.3	Stability and Causality	273
9-9	Using Convolution to Remove Multipath Distortion	276
9-10	Summary	278
9-11	Problems	279

10 Frequency Response 285

10-1	The Frequency Response Function for LTI Systems	285
10-1.1	Plotting the Frequency Response	287
10-1.1.1	Logarithmic Plot	288
10-1.2	Magnitude and Phase Changes	288

10-2	Response to Real Sinusoidal Signals	289
10-2.1	Cosine Inputs	290
10-2.2	Symmetry of $H(j\omega)$	290
10-2.3	Response to a General Sum of Sinusoids	293
10-2.4	Periodic Input Signals	294
10-3	Ideal Filters	295
10-3.1	Ideal Delay System	295
10-3.2	Ideal Lowpass Filter	296
10-3.3	Ideal Highpass Filter	297
10-3.4	Ideal Bandpass Filter	297
10-4	Application of Ideal Filters	298
10-5	Time-Domain or Frequency-Domain?	300
10-6	Summary/Future	301
10-7	Problems	302

11 Continuous-Time Fourier Transform 307

11-1	Definition of the Fourier Transform	308
11-2	Fourier Transform and the Spectrum	310
11-2.1	Limit of the Fourier Series	310
11-3	Existence and Convergence of the Fourier Transform	312
11-4	Examples of Fourier Transform Pairs	313
11-4.1	Right-Sided Real Exponential Signals	313
11-4.1.1	Bandwidth and Decay Rate	314
11-4.2	Rectangular Pulse Signals	314
11-4.3	Bandlimited Signals	316
11-4.4	Impulse in Time or Frequency	317
11-4.5	Sinusoids	318
11-4.6	Periodic Signals	319
11-5	Properties of Fourier Transform Pairs	322
11-5.1	The Scaling Property	322
11-5.2	Symmetry Properties of Fourier Transform Pairs	324
11-6	The Convolution Property	326
11-6.1	Frequency Response	326
11-6.2	Fourier Transform of a Convolution	327
11-6.3	Examples of the Use of the Convolution Property	328
11-6.3.1	Convolution of Two Bandlimited Functions	328
11-6.3.2	Product of Two Sinc Functions	329
11-6.3.3	Partial Fraction Expansions	330

- 11-7 Basic LTI Systems 332
 - 11-7.1 Time Delay 332
 - 11-7.2 Differentiation 333
 - 11-7.3 Systems Described by Differential Equations 334
- 11-8 The Multiplication Property 335
 - 11-8.1 The General Signal Multiplication Property 335
 - 11-8.2 The Frequency Shifting Property 336
- 11-9 Table of Fourier Transform Properties and Pairs 337
- 11-10 Using the Fourier Transform for Multipath Analysis 337
- 11-11 Summary 341
- 11-12 Problems 342

12 Filtering, Modulation, and Sampling 346

- 12-1 Linear Time-Invariant Systems 346
 - 12-1.1 Cascade and Parallel Configurations 347
 - 12-1.2 Ideal Delay 348
 - 12-1.3 Frequency Selective Filters 351
 - 12-1.3.1 Ideal Lowpass Filter 351
 - 12-1.3.2 Other Ideal Frequency Selective Filters 352
 - 12-1.4 Example of Filtering in the Frequency-Domain 353
 - 12-1.5 Compensation for the Effect of an LTI Filter 355
- 12-2 Sinewave Amplitude Modulation 358
 - 12-2.1 Double-Sideband Amplitude Modulation 358
 - 12-2.2 DSBAM with Transmitted Carrier (DSBAM-TC) 362
 - 12-2.3 Frequency Division Multiplexing 366
- 12-3 Sampling and Reconstruction 368
 - 12-3.1 The Sampling Theorem and Aliasing 368
 - 12-3.2 Bandlimited Signal Reconstruction 370
 - 12-3.3 Bandlimited Interpolation 372
 - 12-3.4 Ideal C-to-D and D-to-C Converters 373
 - 12-3.5 The Discrete-Time Fourier Transform 375
 - 12-3.6 The Inverse DTFT 376
 - 12-3.7 Discrete-Time Filtering of Continuous-Time Signals 377
- 12-4 Summary 380
- 12-5 Problems 381

13	Computing the Spectrum	389
13-1	Finite Fourier Sum	390
13-2	Too Many Fourier Transforms?	391
	13-2.1 Relation of the DTFT to the CTFT	392
	13-2.2 Relation of the DFT to the DTFT	393
	13-2.3 Relation of the DFT to the CTFT	393
13-3	Time-Windowing	393
13-4	Analysis of a Sum of Sinusoids	395
	13-4.1 DTFT of a Windowed Sinusoid	398
13-5	Discrete Fourier Transform	399
	13-5.1 The Inverse DFT	400
	13-5.2 Summary of the DFT Representation	401
	13-5.3 The Fast Fourier Transform (FFT)	402
	13-5.4 Negative Frequencies and the DFT	402
	13-5.5 DFT Example	403
13-6	Spectrum Analysis of Finite-Length Signals	405
13-7	Spectrum Analysis of Periodic Signals	407
13-8	The Spectrogram	408
	13-8.1 Spectrogram Display	409
	13-8.2 Spectrograms in MATLAB	410
	13-8.3 Spectrogram of a Sampled Periodic Signal	410
	13-8.4 Resolution of the Spectrogram	411
	13-8.4.1 Resolution Experiment	412
	13-8.5 Spectrogram of a Musical Scale	413
	13-8.6 Spectrogram of a Speech Signal	415
	13-8.7 Filtered Speech	418
13-9	The Fast Fourier Transform (FFT)	420
	13-9.1 Derivation of the FFT	420
	13-9.1.1 FFT Operation Count	421
13-10	Summary and Links	423
13-11	Problems	424
A	Complex Numbers	427
A-1	Introduction	428
A-2	Notation for Complex Numbers	428
	A-2.1 Rectangular Form	428

- A-2.2 Polar Form 429
- A-2.3 Conversion: Rectangular and Polar 430
- A-2.4 Difficulty in Second or Third Quadrant 431
- A-3 Euler’s Formula 431
 - A-3.1 Inverse Euler Formulas 432
- A-4 Algebraic Rules for Complex Numbers 432
 - A-4.1 Complex Number Exercises 434
- A-5 Geometric Views of Complex Operations 434
 - A-5.1 Geometric View of Addition 435
 - A-5.2 Geometric View of Subtraction 436
 - A-5.3 Geometric View of Multiplication 437
 - A-5.4 Geometric View of Division 437
 - A-5.5 Geometric View of the Inverse, z^{-1} 437
 - A-5.6 Geometric View of the Conjugate, z^* 438
- A-6 Powers and Roots 438
 - A-6.1 Roots of Unity 439
 - A-6.1.1 Procedure for Finding Multiple Roots 440
- A-7 Summary and Links 441
- A-8 Problems 441

B Programming in MATLAB 443

- B-1 MATLAB Help 444
- B-2 Matrix Operations and Variables 444
 - B-2.1 The Colon Operator 445
 - B-2.2 Matrix and Array Operations 445
 - B-2.2.1 A Review of Matrix Multiplication 445
 - B-2.2.2 Pointwise Array Operations 446
- B-3 Plots and Graphics 446
 - B-3.1 Figure Windows 447
 - B-3.2 Multiple Plots 447
 - B-3.3 Printing and Saving Graphics 447
- B-4 Programming Constructs 447
 - B-4.1 MATLAB Built-in Functions 448
 - B-4.2 Program Flow 448
- B-5 MATLAB Scripts 448

B-6	Writing a MATLAB Function	448
	B-6.1 Creating A Clip Function	449
	B-6.2 Debugging a MATLAB M-file	451
B-7	Programming Tips	451
	B-7.1 Avoiding Loops	452
	B-7.2 Repeating Rows or Columns	452
	B-7.3 Vectorizing Logical Operations	452
	B-7.4 Creating an Impulse	453
	B-7.5 The Find Function	453
	B-7.6 Seek to Vectorize	454
	B-7.7 Programming Style	454

C Laboratory Projects 455

C-1	Introduction to MATLAB	457
	C-1.1 Pre-Lab	457
	C-1.1.1 Overview	457
	C-1.1.2 Movies: MATLAB Tutorials	457
	C-1.1.3 Getting Started	458
	C-1.2 Warm-up	458
	C-1.2.1 MATLAB Array Indexing	459
	C-1.2.2 MATLAB Script Files	459
	C-1.2.3 MATLAB Sound (optional)	460
	C-1.3 Laboratory: Manipulating Sinusoids with MATLAB	460
	C-1.3.1 Theoretical Calculations	461
	C-1.3.2 Complex Amplitude	461
	C-1.4 Lab Review Questions	461
C-2	Encoding and Decoding Touch-Tone Signals	463
	C-2.1 Introduction	463
	C-2.1.1 Review	463
	C-2.1.2 Background: Telephone Touch-Tone Dialing	463
	C-2.1.3 DTMF Decoding	464
	C-2.2 Pre-Lab	464
	C-2.2.1 Signal Concatenation	464
	C-2.2.2 Comment on Efficiency	465
	C-2.2.3 Encoding from a Table	465
	C-2.2.4 Overlay Plotting	465

- C-2.3 Warm-up: DTMF Synthesis 465
 - C-2.3.1 DTMF Dial Function 466
 - C-2.3.2 Simple Bandpass Filter Design 467
- C-2.4 Lab: DTMF Decoding 468
 - C-2.4.1 Filter Bank Design: dtmfdesign.m 468
 - C-2.4.2 A Scoring Function: dtmfscore.m 469
 - C-2.4.3 DTMF Decode Function: dtmfrun.m 470
 - C-2.4.4 Testing 471
 - C-2.4.5 Telephone Numbers 471
 - C-2.4.6 Demo 472
- C-3 Two Convolution GUIs 473
 - C-3.1 Introduction 473
 - C-3.2 Pre-Lab: Run the GUIs 473
 - C-3.2.1 Discrete-Time Convolution Demo 473
 - C-3.2.2 Continuous-Time Convolution Demo 474
 - C-3.3 Warm-up: Run the GUIs 475
 - C-3.3.1 Continuous-Time Convolution GUI 475
 - C-3.3.2 Discrete Convolution GUI 475
 - C-3.4 Lab Exercises 475
 - C-3.4.1 Continuous-Time Convolution 475
 - C-3.4.2 Continuous-Time Convolution Again 476
 - C-3.4.3 Discrete-Time Convolution 476

D CD-ROM Demos 478

Index 482