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\* Denotes sections in which the mathematics is at a higher level. These sections can be omitted without loss of continuity.

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