# Contents

**Translator's Foreword** ix

**Preface** xi

CHAPTER 1. **Signals and Wavelets**
1.1 What Is a Signal? 1
1.2 The Goals of Signal and Image Processing 2
1.3 Stationary Signals, Transient Signals, and Adaptive Coding Algorithms 4
1.4 Grossmann–Morlet Time-Scale Wavelets 5
1.5 Time-Frequency Wavelets from Gabor to Malvar 6
1.6 Optimal Algorithms in Signal Processing 7
1.7 Optimal Representation, according to Marr 9
1.8 Terminology 10
1.9 Reader's Guide 11

CHAPTER 2. **Wavelets From a Historical Perspective**
2.1 Introduction 13
2.2 From Fourier (1807) to Haar (1909), Frequency Analysis Becomes Scale Analysis 14
2.3 New Directions of the 1930s: Lévy and Brownian Motion 18
2.4 New Directions of the 1930s: Littlewood and Paley 19
2.5 New Directions of the 1930s: The Franklin System 21
2.6 New Directions of the 1930s: The Wavelets of Lusin 22
2.7 Atomic Decompositions, from 1960 to 1980 24
2.8 Strömberg's Wavelets 26
2.9 A First Synthesis: Wavelet Analysis 27
2.10 The Advent of Signal Processing 30
2.11 Conclusions 31

CHAPTER 3. **Quadrature Mirror Filters**
3.1 Introduction 33
3.2 Subband Coding: The Case of Ideal Filters 34
3.3 Quadrature Mirror Filters 36
3.4 The Trend and Fluctuation 38
3.5 The Time-Scale Algorithm of Mallat and the Time-Frequency Algorithm of Galand 39
3.6 Trends and Fluctuations with Orthonormal Wavelet Bases 40
3.7 Convergence to Wavelets 42
3.8 The Wavelets of Daubechies 43
3.9 Conclusions 44

CHAPTER 4. **Pyramid Algorithms for Numerical Image Processing**
4.1 Introduction 45
4.2 The Pyramid Algorithms of Burt and Adelson 46
4.3 Examples of Pyramid Algorithms 50
4.4 Pyramid Algorithms and Image Compression 51
4.5 Pyramid Algorithms and Multiresolution Analysis 53
4.6 The Orthogonal Pyramids and Wavelets 55
4.7 Bi-orthogonal Wavelets 59

CHAPTER 5. **Time-Frequency Analysis for Signal Processing**
5.1 Introduction 63
5.2 The Time-Frequency Plane 66
5.3 The Wigner–Ville Transform 66
5.4 The Computation of Certain Wigner–Ville Transforms 67
5.5 The Wigner–Ville Transform and Pseudodifferential Calculus 69
5.6 The Wigner–Ville Transform and Instantaneous Frequency 70
5.7 The Wigner–Ville Transform of Asymptotic Signals 72
5.8 Return to the Problem of Optimal Decomposition in Time-Frequency Atoms 73

CHAPTER 6. **Time-Frequency Algorithms Using Malvar's Wavelets**
6.1 Introduction 75
6.2 Malvar Wavelets: A Historical Perspective 76
6.3 Windows with Variable Lengths 77
6.4 Malvar Wavelets and Time-Scale Wavelets 80
6.5 Adaptive Segmentation and the Split-and-Merge Algorithm 81
6.6 The Entropy of a Vector with Respect to an Orthonormal Basis 83
6.7 The Algorithm for Finding the Optimal Malvar Basis 83
6.8 An Example Where This Algorithm Works 85
6.9 The Discrete Case 86

CHAPTER 7. **Time-Frequency Analysis and Wavelet Packets**
7.1 Heuristic Considerations 89
7.2 The Definition of Basic Wavelet Packets 92
7.3 General Wavelet Packets 95
7.4 Splitting Algorithms 97
7.5 Conclusions 98

CHAPTER 8. **Computer Vision and Human Vision**
8.1 Marr's Program 101
8.2 The Theory of Zero-Crossings 104
8.3 A Counterexample to Marr's Conjecture 105
8.4 Mallat's Conjecture 106
8.5 The Two-Dimensional Version of Mallat's Algorithm 109
8.6 Conclusions 110

CHAPTER 9. **Wavelets and Fractals**
9.1 Introduction 111
9.2 The Weierstrass Function 111
9.3 The Determination of Regular Points in a Fractal Background 113
9.4 Study of the Riemann Function 115
9.5 Conclusions 117

CHAPTER 10. **Wavelets and Turbulence**
10.1 Introduction 119
10.2 The Statistical Theory of Turbulence and Fourier Analysis 119
10.3 Verification of the Hypothesis of Parisi and Frisch 120
10.4 Farge's Experiments 121
10.5 Numerical Approaches to Turbulence 122

CHAPTER 11. **Wavelets and the Study of Distant Galaxies**
11.1 Introduction 125
11.2 The New Telescopes 125
11.3 The Hierarchical Organization of the Galaxies and the Creation of the Universe 126
11.4 The Multifractal Approach to the Universe 126
11.5 The Advent of Wavelets 126

**Index** 129