Contents

Preface		v
Acknowlegments		xi
I. TH	E PHENOMENOLOGY OF CHAOS	1
1 Ti	ree Chaotic Systems	3
1.1 1.2 1.3	A Nonlinear Electrical System	3 4 9
1.4 1.5 1.6	A Mathematical Model of Biological Population Growth A Model of Convecting Fluids: The Lorenz Model Determinism, Unpredictability, and Divergence of	19 28
1.7 1.8	Trajectories Summary and Conclusions Further Reading	39 41 42
2 The Universality of Chaos		46
2.1 2.2 2.3 2.4 2.5 2.6 2.7	Introduction The Feigenbaum Numbers Convergence Ratio for Real Systems Using δ to Make Predictions Feigenbaum Size Scaling Self-Similarity Other Universal Features	46 46 50 52 54 56 58
2.8 2.9 2.10 2.11	Comments on Models and the Universality of Chaos Computers and Chaos Further Reading Computer Exercises	58 61 62 63
	OWARD A THEORY OF ONLINEAR DYNAMICS AND CHAOS	67

xiv

	namics in State Space: ne and Two Dimensions	69
3.1	Introduction	69
3.2	State Space	70
3.3	Systems Described by First-Order Differential	
	Equations	73
3.4	The No-Intersection Theorem	76
3.5	Dissipative Systems	77
3.6	One-Dimensional State Space	79
3.7	Taylor Series Linearization Near Fixed Points	83
3.8	Trajectories in a One-Dimensional State Space	85
3.9	Dissipation Revisited	87
3.10	Two-Dimensional State Space	88
3.11	Two-Dimensional State Space: The General Case	93
3.12	Dynamics and Complex Characteristic Values	96
3.13	Dissipation and the Divergence Theorem	100
3.14	The Jacobian Matrix for Characteristic Values	100
3.15	Limit Cycles	103 105
3.16	Poincaré Sections and the Stability of Limit Cycles * The van der Pol Oscillator	103
3.17 3.18	Bifurcation Theory	118
3.19	* Example—A Simple Laser Model	126
3.20	Summary	133
3.21	Further Reading	134
3.22	Computer Exercises	135
	ree-Dimensional State Space and Chaos	136
4.1	Overview	136
4.2	Heuristics	137
4.3	Routes to Chaos	140
4.4	Three-dimensional Dynamical Systems	142
4.5	Fixed Points in Three Dimensions	144
4.6	Limit Cycles and Poincaré Sections	148
4.7	Quasi-Periodic Behavior	154
4.8	The Routes to Chaos I: Period-Doubling	157
4.9	The Routes to Chaos II: Quasi-Periodicity	158
4.10	Routes to Chaos III: Intermittency and Crises	159
4.11	The Routes to Chaos IV:	1.00
	Chaotic Transients and Homoclinic Orbits	160
4.12	Homoclinic Tangles and Horseshoes	169
4.13	Lyapunov Exponents and Chaos	171
4.14	Further Reading	178
4.15	Computer Exercises	179

Contents		xv
5 Ite	erated Maps	180
5.1	Introduction	180
5.2	Poincaré Sections and Iterated Maps	181
5.3	One-Dimensional Iterated Maps	187
5.4	Bifurcations in Iterated Maps:	107
5.1	Period Doubling, Chaos, and Lyapunov Exponents	191
5.5	Qualitative Universal Behavior: The <i>U</i> -Sequence	198
5.6	*Theory of the Universal Feigenbaum Number α	210
	[11]	
5.7	*Derivation of the Feigenbaum Number δ	217
5.8	Other Universal Features	221
5.9	Tent Map	225
5.10	Shift Maps and Symbolic Dynamics	228
5.11	The Gaussian Map	234
5.12	Two-Dimensional Iterated Maps	239
5.13	The Smale Horseshoe Map	242
5.14	Summary	247
5.15	Further Reading	247
5.16	Computer Exercises	249
6 Q	uasi-Periodicity and Chaos	252
6.1	Introduction	252
6.2	Quasi-Periodicity and Poincaré Sections	254
6.3	Quasi-Periodic Route to Chaos	256
6.4	Universality in the Quasi-Periodic Route to Chaos	258
6.5	Frequency-Locking	260
6.6	Winding Numbers	261
6.7	Circle Map	263
6.8	The Devil's Staircase and the Farey Tree	272
6.9	Continued Fractions and Fibonacci Numbers	276
6.10	On to Chaos and Universality	280
6.11	Some Applications	285
6.12	Further Reading	292
6.13	Computer Exercises	294
7 In	termittency and Crises	295
7.1	Introduction	295
7.2	What Is Intermittency?	295
7.3	The Cause of Intermittency	297
7.4	Quantitative Theory of Intermittency	301
7.5	Types of Intermittency and	
	Experimental Observations	304
7.6	Crises	306
7.7	Some Conclusions	313
7.8	Further Reading	313

xvi		Contents
7.	9 Computer Exercises	315
8	Hamiltonian Systems	316
8.		316
8	Degations and the Halling like	318
8.4		320
8.5	Integrable Hamiltonians Nonintegrable Systems, the KAM Theorem.	325
8.6	and Period-Doubling	335
8.7		343
8.8	The Arnold Cat Map	351 356
8.9	The Dissipative Standard Map	358
8.1 8.1	Transfer of Transfer Dylidillics	360
8.1		361
	P WELL EMELONS	363
III.	MEASURES OF CHAOS	365
9	Quantifying Chaos	367
9.1 9.2		367
9.3	Series of Dynamical validities	368
9.4	July mile i Zirbonone	371
9.5	Invariant Measure	376 380
9.6	Boto F Billian Lillion	386
9.7	Fractal Dimension(s)	392
9.8	and a strictly of the time of	
9.9	Computational Case History Comments and Conclusions	407
9.1	the Contraction	421
9.1		421 426
10	Many Dimensions and Multifractals	420
10.		
10.2	and mid the odder	427 428
10.3	Practical Considerations for	420
10	Embedding Calculations	436
10.4		
10.5	Generalized Correlation Sums Multifractals and the Spectrum of	443
10	Multifractals and the Spectrum of Scaling Indices $f(\alpha)$	
10.6		448
10.0	Generalized Entropy and the $g(\Lambda)$ Spectrum	462

Contents		xvii
10.7 10.8	Characterizing Chaos via Periodic Orbits *Statistical Mechanical and	472
	Thermodynamic Formalism	475
10.9	Summary	481
10.10	Further Reading	482
10.11	Computer Exercises	485
IV. SP	ECIAL TOPICS	487
11 Pa	ttern Formation and Spatiotemporal Chaos	489
11.1	Introduction	489
11.2	Two-Dimensional Fluid Flow	491
11.3	Coupled-Oscillator Models and Cellular Automata	499
11.4	Transport Models	507
		307
11.5	Reaction-Diffusion Systems:	520
11.6	A Paradigm for Pattern Formation	520
11.6	Diffusion-Limited Aggregation, Dielectric Break-	700
	down, and Viscous Fingering: Fractals Revisited	533
11.7	Self-Organized Criticality: The Physics of Fractals	541
11.8	Summary	542
11.9	Further Reading	543
11.10		548
	antum Chaos, The Theory of Complexity, and	5.40
Ot	her Topics	549
12.1	Introduction	549
	Quantum Mechanics and Chaos	549
12.3	Chaos and Algorithmic Complexity	569
12.4	Miscellaneous Topics: Piece-wise Linear Models,	307
	Time-Delay Models, Information Theory, Com-	
	puter Networks, and Controlling Chaos	572
12.5	Roll Your Own: Some Simple Chaos Experiments	578
12.6	General Comments and Overview:	
	The Future of Chaos	579
12.7	Further Reading	581
Append	ix A: Fourier Power Spectra	589
* *	ix B: Bifurcation Theory	599
	ix C: The Lorenz Model	605
	ix D: The Research Literature on Chaos	619
	ix E: Computer Programs	620
Referer	ices	628
Index		649
A C. C.		